CALCULATION OF THE DIFFUSION COMBUSTION OF A SUBSONIC JET IN A CO-CURRENT SUPERSONIC FLOW

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An effective method of reducing base drag on aircraft in supersonic flight regimes is organizing combustion in their afterbody by the additional injection of fuel through the base. The practical use of this method of controlling base drag makes it important to consider the question of allowing for the effect of combustion on the characteristics of the flow field in the neighborhood of the body. The study [1] examined the combustion of a supersonic injected jet in an afterbody on the basis of the Navier–Stokes equations. The effect of diffusion combustion on the characteristics of separated base flows was examined in [2] within the framework of the Chapman–Korst model.

Here we propose an approximate method of calculating the parameters of an injected subsonic jet in a co-current supersonic flow in the presence of diffusion combustion. The method is based on a model of the strong viscous-inviscid interaction, through pressure, of conjugate inviscid flows with a flow in the viscous region.

1. Formulation of the Problem and Method of Solution. We are examining a steady flow in the two-dimensional wake behind a body. We will use the same formulation as in [3] but with allowance for the diffusion combustion of a subsonic injected jet in a supersonic co-current flow. In regard to the interacting flows, it is assumed that they each contain one reactive component (an oxidant R for the external flow, a fuel A for the injected jet) and N components which are nonreactive under the given conditions. When mixed in the mixing region and the near wake, the oxidant and fuel enter into a chemical reaction – combustion. The latter is described by a reaction of the form

$$\nu_R R + \nu_A A \to \nu_S S,$$

where S is the reaction product; ν_i is the stoichiometric coefficient of the i-th component. In the given combustion model, the reaction is localized in an infinitely narrow region – the flame front [4]. In the mathematical description, this front corresponds to a surface of discontinuity of the heat flows and diffusion flows of the reacting components. Use of the model of diffusion combustion appreciably simplifies the calculation, since in this case it is unnecessary to introduce chemical kinetics and chemical equilibrium constants.

Features of the problem as formulated include the formation of two flame fronts in the aft region of the body in question (in the general case, these fronts are asymmetrical). The fronts are directed counter to each other and disappear when they meet due to the combustion of the fuel in the jet. The presence of two flame fronts in the viscous region divides it into three subregions. In the flow region between the fronts, component R is absent. Outside this region, component A is absent. The viscous flow is approximately described by the equations of a multicomponent, reactive, laminar boundary layer [5], augmented by the equation of state and the dependences of the transport coefficients on temperature and the composition of the mixture. The solutions in the respective subregions of viscous and inviscid flow are joined together by means of viscous–inviscid interaction conditions [3]. These conditions, generalized to the case of chemical reaction and the presence of closed separation zones in the viscous region, have the form

$$dp_e/dx = (A + B + C)/D, \tag{1.1}$$

$$\frac{dy_k^*}{dx} = q_k, \quad \frac{dq_k}{dx} = \frac{1+q_k^2}{\gamma_k M_k^2 p_e}\left(q_k \frac{A+B+C}{D} - \frac{\partial p}{\partial y}\Big|_{y=y_k^*}\right), \quad k = 1, 2,$$

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where A, B, C, and D are determined as follows:

$$A = p_e\left(u_k^2 y_k^{*j}\frac{dy_k^*}{dx}\Big|_{k=1}^{k=2} - 2\int_{y_1}^{y_2} v\frac{\partial u}{\partial y}y^j dy\right), \tag{1.2}$$

$$B = \int_{y_1}^{y_2}\frac{\gamma-1}{\gamma}u\left\{\frac{\partial}{\partial y}\left(y^j\frac{\mu}{\mathrm{Pr}}\frac{\partial h}{\partial y}\right) + y^j\mu\left(\frac{\partial u}{\partial y}\right)^2\right\}dy - \int_{y_1}^{y_2}\frac{p_e}{\rho}\frac{\partial}{\partial y}\left(y^j\mu\frac{\partial u}{\partial y}\right)dy,$$

$$C = \int_{y_1}^{y_2}\frac{\gamma-1}{\gamma}u\left\{\frac{\partial}{\partial y}\left[y^j\sum_i\frac{\mu}{\mathrm{Pr}}(\mathrm{Le}_i-1)h_i\frac{\partial c_i}{\partial y}\right] - \sum_i\left(h_i - \frac{\varkappa}{\varkappa_i}\frac{a^2}{\gamma-1}\right)\frac{\partial}{\partial y}\left(y^j\frac{\mu}{\mathrm{Sm}_i}\frac{\partial c_i}{\partial y}\right)\right\}dy,$$

$$D = \frac{1}{1+j}\left[\int_{y_1}^{y_2}\frac{\partial}{\partial y}\left(\frac{u^2-a^2}{\gamma}\right)y^{1+j}dy - \frac{u_k^2-a_k^2}{\gamma_k}y_k^{*1+j}\Big|_{k=1}^{k=2}\right].$$

Here, c_i is the mass concentration of the i-th component; $\varkappa_i$ and $\varkappa$ are the molecular weights of the component and the mixture; Sm and Le are the Schmidt and Lewis numbers; a is the speed of sound (the rest of the notation was taken from [3]).

System (1.1) was obtained from the law of mass conservation in the viscous flow region and the boundary conditions for the transverse velocity component and the coupled flows. Within a broad range of the initial data, system (1.1) has a saddle point when integrated together with the Euler equations and boundary-layer equations. The presence of the saddle point, which manifests the upstream mechanism of transmission of perturbations, is a consequence of the slight ellipticity of given problem in the viscous–inviscid interaction model. The ellipticity is due to the fact that the problem involves unknown boundaries. Physcially, the flow in question corresponds to the singular integral curve of system (1.1). Thus, the viscous–inviscid interaction conditions allow us to find the distribution of static pressure in the viscous region, construct the boundaries of the coupled inviscid flows, and connect the solutions in the respective subregions.

Calculation of the parameters in the near wake behind the body involves simultaneous stepwise integration of the Euler equations, the equations of the boundary layer, and Eqs. (1.1). We use an explicit finite-difference scheme [6] to calculate the characteristics of the equivalent inviscid flows, while the solution of the equations of viscous–inviscid interaction (1.1) is obtained by the Runge–Kutta method. Some difficulties are encountered in integration of the equations of a reactive laminar boundary layer. These difficulties have to do, on the one hand, with the asymptotic character of the boundary conditions, the need to determine the boundaries of the viscous region during the calculations, and the considerable nonuniformity of the parameters across the flow. On the other hand, the difficulties are also related to the formation of diffusion flame fronts in the viscous flow region. The boundary-layer equations are integrated by an implicit finite-difference scheme in normalized von Mises variables [7]:

$$\xi = x,\ \eta = (\Psi - \Psi_1(x))/(\Psi_2(x) - \Psi_1(x)),$$

where Ψ is the stream function; $\partial\Psi/\partial y = y^j\rho u$; $\partial\Psi/\partial x = -y^j\rho v$; $\Psi_k(x)$ is the value of the stream function on the corresponding boundary of the viscous region $y = y_k(x)$. Use of the von Mises variables reduces the number of governing functions, avoids the problems connected with satisfaction of the boundary conditions, and automatically condenses the nodes of the computing grid in the physical plane in the neighborhood of the boundaries of the viscous region. The latter fact makes it possible to more accurately account for the mass brought into the viscous region due to viscous drag and to perform calculations with a moderate grid number ($\sim$50). The unknown values of the stream functions at the nominal boundaries of the viscous region are determined from the solution of the system of ordinary differential equations

$$\frac{d\Psi_k}{dx} = -\frac{1}{\Psi_2-\Psi_1}\lim_{\eta\to k-1}\frac{\frac{\partial}{\partial\eta}\left[\mu\rho u y^{2j}\frac{\partial u}{\partial\eta}\right]}{\frac{\partial u}{\partial\eta}} \quad \text{at} \quad k = 1, 2.$$

In accordance with the chosen model of diffusion combustion, the concentration of reactive components is zero at the flame front, while their diffusion flows are in a stoichiometric ratio. Also, the heat flows undergo discontinuities at the flame front due to the liberation (absorption) of heat in the chemical reaction [4]. Having excluded the concentration of the reactive components by means of the reation $c_{A,R} = 1 - c_S - \sum_{i=1}^{N} c_i$, these conditions are conveniently written in the form

$$c_S + \sum_{i=1}^{N} c_i = 1; \tag{1.3}$$

$$\left[\frac{1}{\mathrm{Sm}_A}\frac{\partial c_S}{\partial \eta}\right]_{\eta_{fk}\pm} + L\left[\frac{1}{\mathrm{Sm}_R}\frac{\partial c_S}{\partial \eta}\right]_{\eta_{fk}\mp} = \left\{\left[\frac{1}{\mathrm{Sm}_A}\right]_{\eta_{fk}\pm} + \left[\frac{1}{\mathrm{Sm}_R}\right]_{\eta_{fk}\mp}\right\}\sum_{i=1}^{N}\frac{\partial c_i}{\partial \eta}; \tag{1.4}$$

$$\left[\frac{\partial H}{\partial \eta} + (\mathrm{Le}_S - 1)h_S\frac{\partial c_S}{\partial \eta}\right]\Bigg|_{\eta_{fk}-}^{\eta_{fk}+} = 0. \tag{1.5}$$

Here, the flame front corresponds to the normalized coordinate $\eta = \eta_{fk}(x)$; $L = \nu_A\varkappa_A/\nu_R\varkappa_R$.

Conditions (1.3) and (1.5) serve as boundary conditions for the concentrations of the reactive components and the total enthalpy at the flame front. Equation (1.4) is used to determine the position of the front. The presence of surfaces of discontinuity of the diffusion and heat flows in the theoretical region significantly complicates calculation of the characteristics of the viscous flow. One of the methods widely used to calculate discontinuities is their isolation. In regard to the problem being examined, this approach consists of isolating the flame fronts in the theoretical region, which in turn involves its subdivision into three subregions containing only one component each. Here, to integrate the boundary-layer equations across the viscous region in the presence of a flame front, we resort to the method of "floating" discontinuity. This method allows us to calculate the flow field without a substantial change in the logic of the "through" computing algorithm [3] while at the same time following the position of the front. Let us take a detailed look at the organization of the calculation in this case. At the first stage of the calculations, we assume that the position of the flame front in the section $x = x_n$ coincides with the m-th interior grid node. In solving the diffusion equations for the reactive components by the method of scalar trial run, the correction factors for the calculation of total enthalpy at the front are found from difference analogs of Eq. (1.5) and the energy equation, written at the (m − 1)-st and (m + 1)-st nodes. The correction factors at the remaining nodes are determined in the usual manner. The resulting concentrations of the reactive components are used to calculate the error $\Delta\Phi_m$ in the relation between diffusion flows obtained due to finite-difference approximation (1.4). The number of the node m is chosen on the basis of the condition $\Delta\Phi_m\Delta\Phi_{m+1} < 0$. Here, the position of the flame front turns out to be associated with the error $O(\Delta\eta)$. At the second stage of the calculations, to refine the position of the flame front we introduce the parameter $\Delta\eta_f = \eta_f - \eta_m$. The value of this parameter is found from $\Delta\Phi_f = 0$. As before, Eqs. (1.3) and (1.5) serve to determine the correction factors at the m-th node. For example, using linear interpolation, from (1.3) we obtain

$$(1 + t_f)c_{S_m} - t_f c_{S_{m-1}} = 1 - \varphi_f, \tag{1.6}$$

where $t_f = \Delta\eta_f/\Delta\eta$; $\varphi_m = \sum_{i=1}^{N} c_{i_m}$; $\varphi_f = (1 - t_f)\varphi_m + t_f\varphi_{m+1}$. It is easy to use (1.6) to obtain the correction factors at the m-th node for the concentrations. Difference approximation of Eqs. (1.4) and (1.5) with $y = y_m + \Delta y_f$, using a linear interpolation of type (1.6), makes it possible to find the correction factors for total enthalpy at the m-th node and to determine $\Delta\Phi_f$.

2. Some Results of the Calculations. Within the framework of the viscous-inviscid interaction model, we evaluated the effect of chemical nonuniformity and diffusion combustion on the characteristics of laminar flow behind a plate in a supersonic flow in the presence of subsonic aft injection. As the external flow, we examined air. The latter was modeled in the calcuations by a two-component mixture of gases containing 23% O_2 and 77% N_2. We initially studied the effect of the molecular weight of the injected jet on the distribution of static pressure in the near wake. The subsonic jet was assumed to consist of one com-

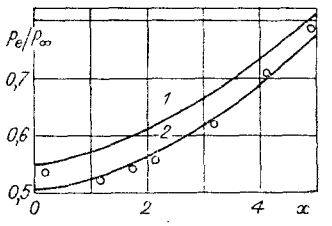

Fig. 1

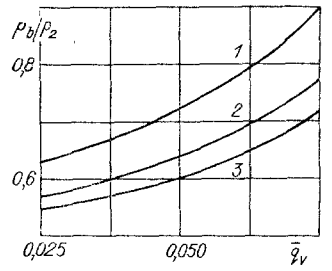

Fig. 2

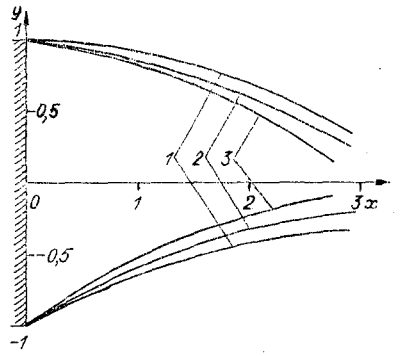

Fig. 3

ponent and was assigned an intensity of uniform injection equal to $\bar{q}_V = \rho_V u_V/\rho_\infty u_\infty$ in the cross section of the aft region of the body. In the calculations, the dependences of the transport coefficients of the pure components on temperature were determined from the power law in [8], while approximate relations in [8] were used to find the same coefficients for the mixture and the generalized diffusion coefficients. Figure 1 shows theoretical distributions of static pressure in the viscous region in the case of a plate in a symmetrical uniform external flow with $M_\infty = 3.5$ and $\bar{q}_V = 0.02$. Calculated curves 1 and 2, corresponding to the injection of helium and nitrogen, agree satisfactorily with the experimental data in [9] (shown by points).

We studied the effect of the diffusion flow of hydrogen on base pressure. The jet was assumed to consist of three components. We chose hydrogen as the reactive component, while the nonreactive components under the given conditions were molecular nitrogen and water vapor. Figure 2 shows the effect of $\bar{q}_V$ on base pressure in nonsymmetrical flow about the plate. The calculations were performed for a cold jet ($T_V^0 = T_1^0 = T_2^0$, $T_1 = T_2 = 300$ K) with the Reynolds number calculated from the parameters of the undisturbed flow and the plate width (Re = 500), the Mach numbers in the bottom and top flows ($M_1 = 2$, $M_2 = 4$), and static pressure in the undisturbed inviscid flows $p_1 = p_2 = 3.039 \cdot 10^4$ N/m^2. The initial and boundary values of the concentrations of the components in the jet: $c_{N_2} = 0.5$, $c_{H_2O} = 1 - c_{N_2} - c_{H_2}$. Curves 1-3 correspond to $c_{H_2} = 0.1$; 0.05; 0.01. The positions of the flame fronts in the viscous region for the present example are shown in Fig. 3. Analysis of the results permits the following conclusions. An increase in the concentration of hydrogen in the jet has a significant effect on base pressure and leads to its increase within the investigated range of injection intensity. With an increase in hydrogen concentration, each of the flame fronts is shifted across the viscous region in the direction of the respective boundary.

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